Analysis of printed structures including thick slots

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Abstract

A full wave method to analyze slots in thick conducting screens is presented. The method is partially based on the fact that spectral-domain transmission line Green's Functions for a multilayered structure (and in particular for a parallel-plate structure) are closed-form integrable along the axis normal to the stratification. This property is exploited to obtain a fast, full wave mixed spectral/spatial Method of Moments (MoM) formulation. For electric elements, a mixed potential formulation is employed. Interaction between electric and magnetic elements is handled with a field formulation. The advantages of these choices are briefly discussed. Two examples are discussed; comparisons with measurements and alternative approaches are presented.

1 Motivation

The finite thickness of metallization forming patches and ground planes (in which slots may be cut) is usually neglected at microwave frequencies. However, this is no longer possible at millimeter and sub-millimeter wavelengths, if an accurate modeling is desired.

Patches of non-zero thickness have already been analyzed by either approximate or full-wave methods (the latter involving a complete mesh of the upper and lower surfaces of the patch, and possibly of the walls).

For slots in a ground plane of finite thickness, an approximate method (termed the 'Delta' approach) is discussed in [5]. This method has a great advantage in that the computational burden is not increased with respect to the zero-thickness case. However, it only provides a first order approximation, and, more importantly, it neglects the effect of the walls, particularly the possible coupling between the walls at both sides of the slot. This effect is important in millimetric CPW (coplanar waveguide) circuits that use compact filters [6] or simple lines [1]. There is the additional interest of having a full wave method that can be used as a standard for the evaluation of approximate methods. For such use, the method must be able to analyze arbitrarily shaped slots.

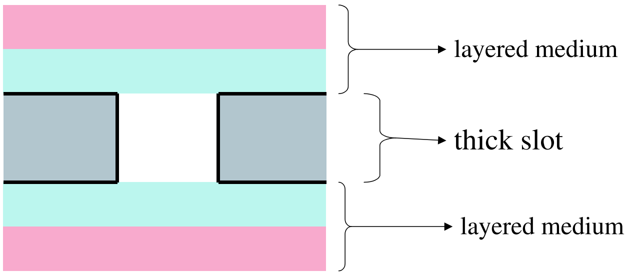

Figure 1: Schema of the problem to solve.

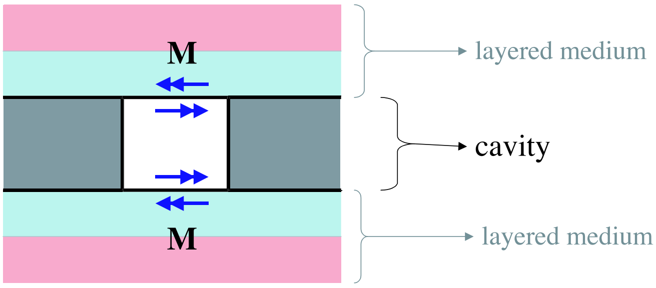

Figure 2: Equivalent problem with cavity or closed waveguide.

2 Equivalent problem

We start by considering the generic problem of figure 1, where a stratified medium includes a thick ground plane in which a slot has been created.

Field equivalence principles can be used to reduce the

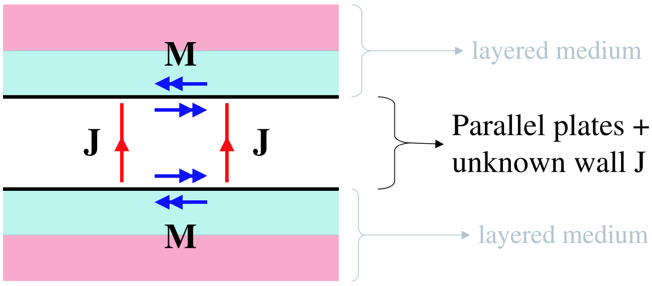

Figure 3: Equivalent problem with parallel plate medium and wall electric current unknowns.

thick slot problem in figure 1 to more tractable equivalent problems by using at least two different strategies.

The first one, shown in figure 2, divides the problem in three parts linked by magnetic currents. In layered media above and below the thick slot, standard method-of-moments integral equation approaches may be used [4]. The medium representing the slot is now a cavity (or a waveguide closed at both ends); here, apropiate modal expansions or cavity/waveguide Green's functions can be used. This method has two main inconvenients. Firstly, the modes of the waveguide must be computed. This is difficult to do for a waveguide of arbitrary section, and may be inaccurate for very complex sections (like a CPW filter). Secondly, the modal expansion must be matched at the apertures to the multilayered media above and below. The procedure requires the computation of extra coupling integrals. For this two reasons, this method is neither simple nor fast, particularly for arbitrarily shaped slots.

The second strategy for creating an equivalent problem, illustrated in figure 3, divides the problem in three parts also, and the media above and below are the same. However, in the medium representing the thick slot (the 'internal' medium), the equivalent is now a parallel plate medium. To account for the metallic walls, unknown electric currents are added. This is the approach taken in this work.

3 Formulation

It is apparent that in either of these methods, the computational burden will at least double when compared to a zero-thickness analysis, owing to the slot aperture being meshed twice. Then there is the interior problem, where the Green's function does not have closed form in the spatial domain. To avoid cumbersome three-dimensional spatial interpolation during the matrix-of-moments fill procedure, part of the integration is effected in the spectral domain. Here, the Green's function has exponential dependence on both source (z') and observer (z) vertical coordinates. Therefore, if the basis functions expansion used for the electric currents of the walls is separable in vertical and horizontal coordinates, vertical integration can be transferred into the spectral domain, performed analytically, and the result stored as a table in the spatial domain. In addition to avoiding three-dimensional interpolation, this procedure has four advantages:

- Integrated Green's functions have better spectral behavior and are thus easier to evaluate in the spatial domain.
- Tables may be reused for differently shaped slots, as long as the vertical structure (on which the spectral integration depends) remains unchanged.
- Spatial integration becomes simpler, not only because surface integrals are reduced to line integrals, but also because they exhibit weaker singularities at the origin ($\mathbf{r}' = \mathbf{r}$) than their non-integrated counterparts.
- Since vertical integrations are now analytical (i.e. exact save for roundoff error), accuracy is increased.

The method is based on the work done in [2] and is related to that presented in [7], although a pure mixed potential formulation has been kept for the electric elements. For the case of the parallel plate medium, this choice leads to less integral types and consequently to reduced memory requirements and faster computations. For the electric-magnetic interactions, a field formulation is used. The singularity of the mixed-type Green's functions $G^{xx}_{\mathbf{EM}}$, $G^{xy}_{\mathbf{EM}}$, $G^{zx}_{\mathbf{EM}}$ and $G^{zy}_{\mathbf{EM}}$ is integrated over the source cell (which is always the magnetic cell) with a closed formula.

4 Examples

Two different examples, a rectangular slot and a dogbone slot antenna, have been analyzed, built and measured. Since the main goal of this work was to analyze the effect of slot thickness, for each example a series of breadboards has been built, where slot thickness varies from 35 μm (printer slot, essentially a zero thickness case) to several mm.

The rectangular slot antenna is fed by coupling to a 50 Ω microstrip line. Its dimensions are 25 mm × 5 mm and it is found to resonate at 6.32 GHz on $h = 0.635$ mm, $\epsilon_r = 10.5$ substrate, for the case where metal thickness equals 35 μm.

Figure 4 shows the mesh used in the MoM analysis, with triangular cells for the magnetic currents in the slot and rectangular cells for the electric currents in the metallic vertical walls. This choice is justified by the fact that, opposite to the vertical walls, slot geometry can be rather complex.

In figure 5 the series of measured breadboards is depicted, with slot thicknesses of ≈ 0, 1, 3, 6 and 10 mm. Comparison between different numerical models and measurements are given in figures 6 to 10. For the 'thick' cases (1 to 10 mm, three different theoretical predictions are shown:

- the 'Delta' function approach [5]
- a rigorous cavity model [3]
- the technique described in this paper.

It can be seen that our technique always follows closely the cavity results (while avoiding the cavity problem) up to $\lambda/5$-thick slots, and that both agree very well with measurements. As expected, the 'Delta' approach is only good for thin slots (up to $\lambda/10$).

It can also be seen that even for slot thickness as small as $\lambda/50$, the effect on resonant frequency is clearly non-negligible (a +3.5% displacement) which stresses the interest and need of this analysis.

The cavity method could only be used for this case.

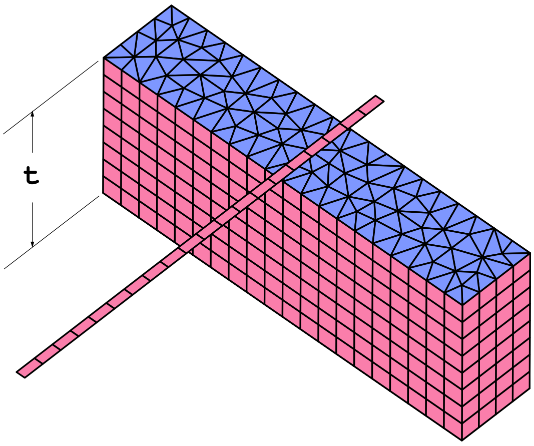

Figure 4: Mesh for the rectangular slot antenna with thickness = 10 mm, showing feed line, slot wall and upper aperture. Electric unknowns red, magnetic unknowns blue. The mesh of the lower aperture is hidden in this view.

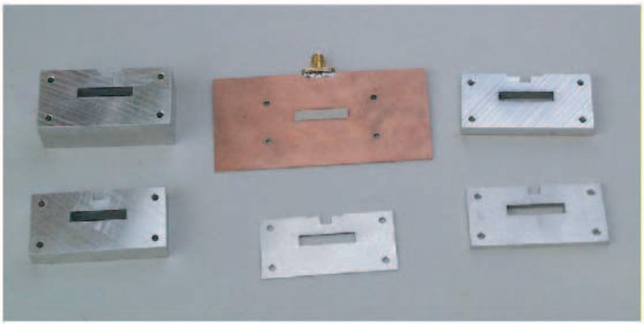

Figure 5: The basic rectangular slot breadboard with metallic masks of various thicknesses.

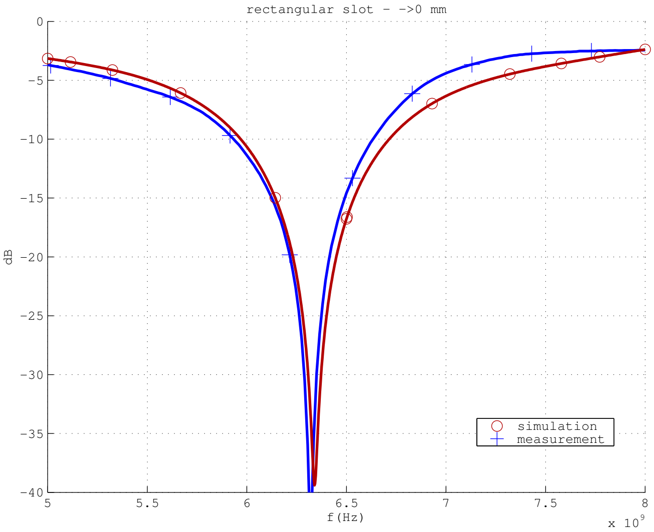

Figure 6: Zero thickness slot (in fact, 35 μm ≈ $\lambda/1400$ @6 GHz).

The second model is a dogbone-shaped slot antenna, also fed by coupling to a 50 Ω microstrip line. Its dimensions are (roughly) 40 mm × 15 mm. It is found to resonate at 4.9 GHz on $h = 0.51$ mm, $\epsilon_r = 2.33$ substrate, for the thickness = 35 μm case. A diagram of the mesh is shown in 11. The freely available mesh generator, TRIANGLE [1], was used. It is worth pointing out that a shape like the dogbone does not add any complexity to our approach, while the cavity approach would become very cumbersome (almost intractable) and, indeed, it has not been considered here.

A photograph of the breadboard series, showing the printed slot antenna and metallic masks of thickness 1 and 3 mm, is shown in figure 12. Simulations and measurements are compared in figures 13 to 15. It can be seen that the numerical analysis follows closely the behavior indicated by the measurements, as thickness increases.

5 Conclusions

A full-wave method to analyze slots in finite metallization screens has been described. This is done by solving

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[1] `http://www-2.cs.cmu.edu/ quake/triangle.html`

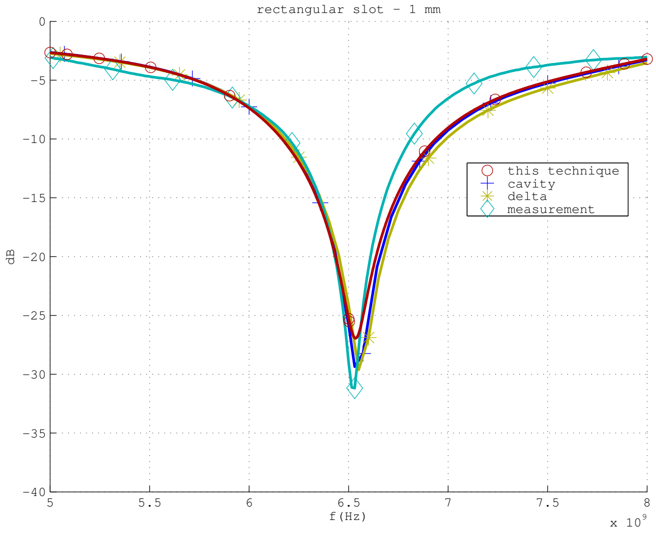

Figure 7: Slot of thickness ≈ $\lambda/50$ @6 GHz.

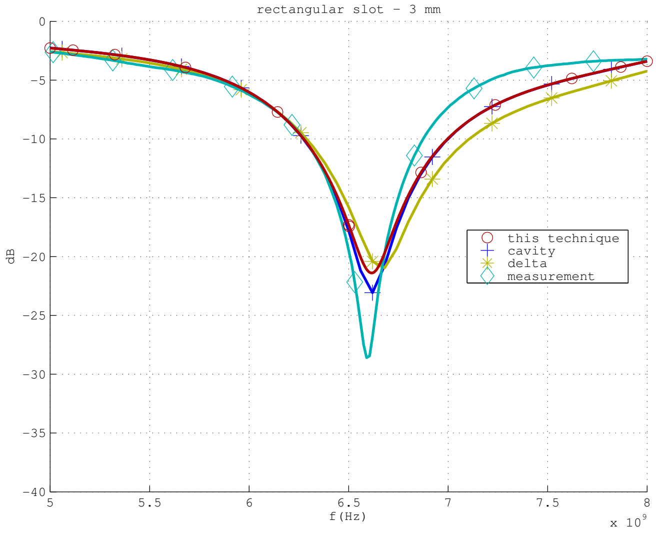

Figure 8: Slot of thickness ≈ $\lambda/15$ @6 GHz.

for magnetic currents at both sides of the slot and for electric currents at the walls on its contour. The problem posed by these vertical walls inside a parallel-plate medium is handled with a mixed spectral-spatial formulation. For a regular shape (rectangular), our method has been validated by comparison with a traditional cavity approach. However the method presented here has the required flexibility to go beyond regular slot shapes, where the use of the cavity approach would be much more involved. Comparison with measurements shows very good agreement. Additionally, the range of applicability of the approximated 'Delta' method [5] has been shown.

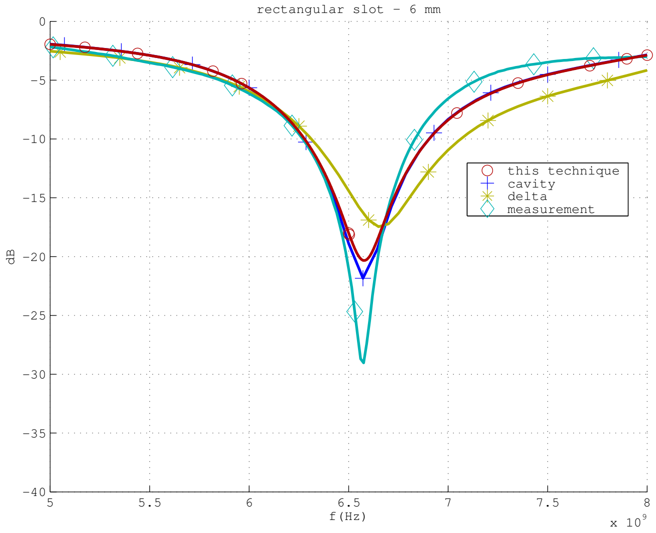

Figure 9: Slot of thickness ≈ $\lambda/8$ @6 GHz.

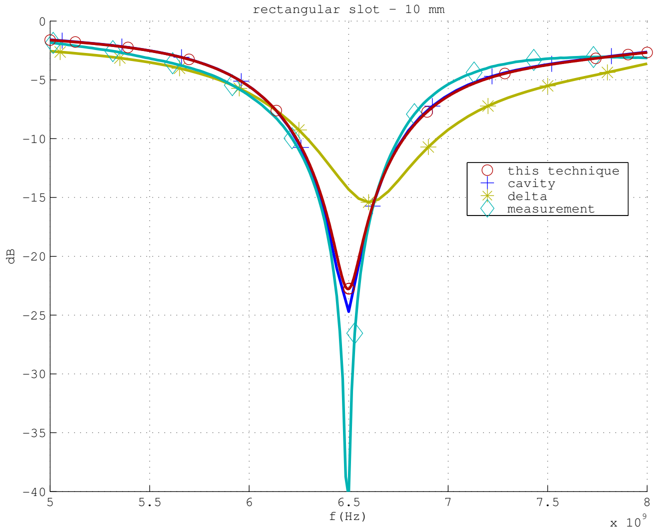

Figure 10: Slot of thickness ≈ $\lambda/5$ @6 GHz.

References

[1] Michael W. Chapman and Sanjay Raman. A 60 GHz uniplanar MMIC 4× subharmonic mixer. *IEEE Trans. on Microwave Theory Tech.*, 50(11):2580–2588, November 2002.

[2] Tomasz Grzegorczyk. *Integrated 3D Antennas for Millimeter-Wave Applications: Theoretical Study and Technological Realization.* PhD thesis, Ecole Polytechnique Fédérale de Lausanne, LEMA-DE, 2000. no. 2299.

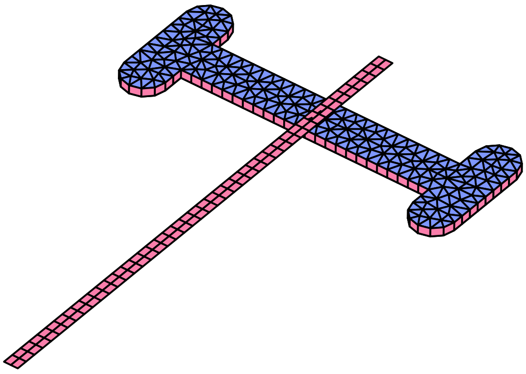

Figure 11: Mesh for the dogbone antenna with thickness = 1 mm.

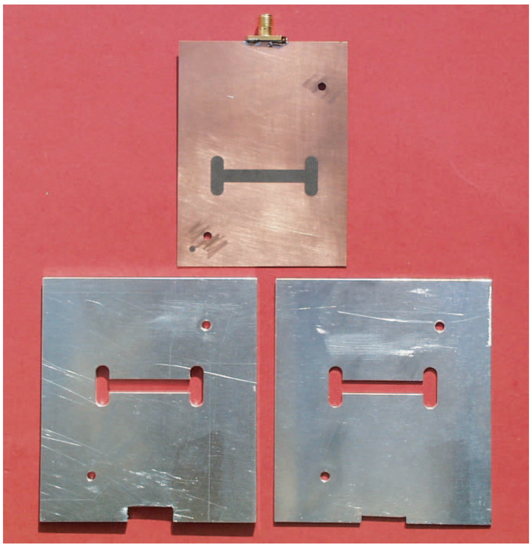

Figure 12: The basic dogbone slot antenna breadboard with metallic masks of two thicknesses.

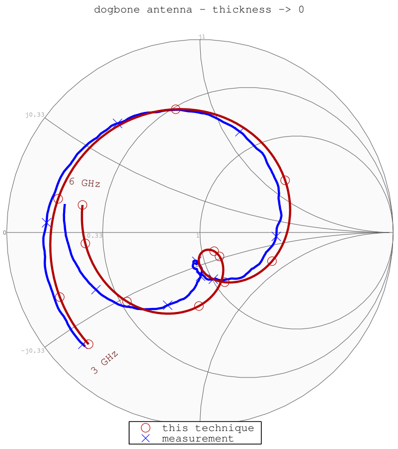

Figure 13: Zero thickness slot (in fact, 35 μm ≈ $\lambda/1700$ @5 GHz).

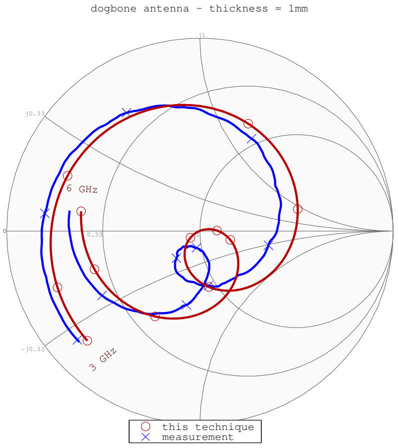

Figure 14: Slot of thickness ≈ $\lambda/60$ @5 GHz.

[3] Alejandro Álvarez Melcón. *Applications of the Integral Equation Technique to the analysis and Synthesis of Multilayered Printed Shielded Microwave Filters and Cavity Backed Antennas.* PhD thesis, Ecole Polytechnique Fédérale de Lausanne, LEMA-DE, 1998. no. 1901.

[4] Juan R. Mosig. Arbitrarily shaped microstrip structures and their analysis with a mixed potential integral equation. *IEEE Trans. on Microwave Theory Tech.*, 36(2):314–323, February 1988.

[5] Juan R. Mosig. Approximate integral equation formulation for thin slots. In *Proceedings of the COST-284 Meeting April 2003, Budapest*, pages 0–0, 2003.

[6] James Sor, Yongxi Qian, and Tatsuo Itoh. Miniature low-loss CPW periodic structures for filter applications. *IEEE Trans. on Microwave Theory Tech.*, 49(12):2336–2341, December 2001.

[7] Mark Vrancken and Guy A. E. Vandenbosch. Hybrid dyadic-mixed potential and combined spectral-space domain integral-equation analysis of quasi 3-d structures in stratified media. *IEEE Trans. on Microwave Theory Tech.*, 51(1):216–225, January 2003.

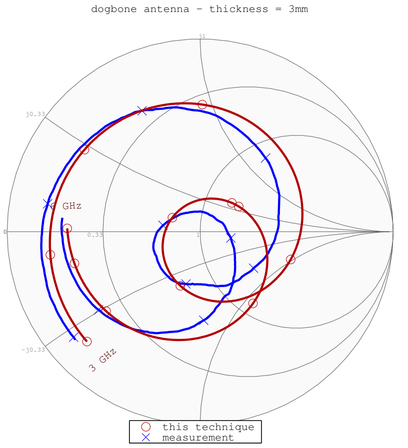

Figure 15: Slot of thickness $\approx \lambda/20$ @5 GHz.